1a) Matrix A is given by

$$\boldsymbol{A} = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{pmatrix},$$

compute the determinant of the matrix, and decide if the vectors $\mathbf{u} = (2, 1, 0)$, $\mathbf{v} = (1, 1, 1)$ and $\mathbf{w} = (1, 2, 3)$ (the rows of the matrix) are linearly independent or linearly dependent.

1b) For two matrices $\boldsymbol{A} = \begin{pmatrix} 2 & -1 \\ 4 & -1 \end{pmatrix}$, $\boldsymbol{B} = \begin{pmatrix} -1 & 3 \\ 0 & 1 \end{pmatrix}$.

Find the inverse matrix $\boldsymbol{A}^{-1}$ and compute matrix $\boldsymbol{X}$ given by matrix equation $\boldsymbol{A} \cdot \boldsymbol{X} = \boldsymbol{B}$.

a) detA $=$ 0 vectors are linearly independent and $\boldsymbol{X} = \begin{pmatrix} 1/2 & -1 \\ 2 & -5 \end{pmatrix}$

b) detA $=$ 0 vectors are linearly dependent and $\boldsymbol{X} = \begin{pmatrix} 1/2 & -1 \\ 2 & -5 \end{pmatrix}$

c) detA $=$ 3 vectors are linearly independent and $\boldsymbol{X} = \begin{pmatrix} 1/2 & 1 \\ -2 & -4 \end{pmatrix}$

d) detA $=$ -1 vectors are linearly independent and $\boldsymbol{X} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2) Find equation of the tangent line to the graph of function $f(x) = \sqrt{x} - \frac{x^2}{2}$ at point $[x_0, f(x_0)]$, if $x_0 = 1$.

a) $y = 2x + 3$ *b)* $y = x - 2$

c) $y = 1 - 1/2x$ *d)* $y = 4 - 2x$.

3 Find the antiderivative (compute indefinite integral) F to function $f(x) = 3 \ \sin(2x)$ and then compute the area bounded by the graph of this function and x-axis on interval $x \in \langle 0, \frac{\pi}{2} \rangle$.

a) $F(x) = 2\sin(2x)$ and area is 3 *b)* $F(x) = 6\cos(2x)$ and area is 2

c) $F(x) = 2\sin(2x)$ and area is 0 *d)* $F(x) = -\frac{3}{2}\cos(2x)$ and area is 3.

Solution:

1a) det $\boldsymbol{A}$ = 0 computed by either Sarus's rule (or by extension along a row or a column). Which implies that the rows are linearly dependent.

$$\text{detA} = \begin{vmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = 2\cdot1\cdot3 + 1\cdot2\cdot0 + 1\cdot1\cdot1 - 0\cdot1\cdot1 - 1\cdot2\cdot2 - 3\cdot1\cdot1 = 0.$$

1b)

$$\boldsymbol{A} = \begin{pmatrix} 2 & -1 \\ 4 & -1 \end{pmatrix} \to \begin{pmatrix} 2 & -1 & | & 1 & 0 \\ 4 & -1 & | & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 2 & -1 & | & 1 & 0 \\ 0 & 1 & | & -2 & 1 \end{pmatrix}$$

$$\sim \begin{pmatrix} 2 & 0 & | & -1 & 1 \\ 0 & 1 & | & -2 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & | & -1/2 & 1/2 \\ 0 & 1 & | & -2 & 1 \end{pmatrix} \to$$

$$\to \begin{pmatrix} -1/2 & 1/2 \\ -2 & 1 \end{pmatrix} = \boldsymbol{A}^{-1}$$

$$\boldsymbol{A}\cdot\boldsymbol{X} = \boldsymbol{B} \;\to\; \boldsymbol{A}^{-1}\cdot\boldsymbol{A}\cdot\boldsymbol{X} = \boldsymbol{A}^{-1}\cdot\boldsymbol{B} \;\to\; \boldsymbol{X} = \boldsymbol{A}^{-1}\cdot\boldsymbol{B}$$

$$\boldsymbol{X} = \begin{pmatrix} 1/2 & -1 \\ 2 & -5 \end{pmatrix}$$

2)

$$f(x) = \sqrt{x} - \frac{x^2}{2} \;\to\; f'(x) = \frac{1}{2\sqrt{x}} - x \text{ and } f(1) = \frac{1}{2},\; f'(1) = -\frac{1}{2}$$

tangent line is $y = \frac{1}{2} - \frac{1}{2}(x-1) = 1 - \frac{1}{2}x.$

3)

$$\int 3\,\sin(2x)\,\mathrm{d}x \;=^{1} \int \frac{3}{2}\,\sin(t)\,\mathrm{d}t = -\frac{3}{2}\cos t + C = -\frac{3}{2}\cos 2x + C$$

$$\int_0^{\frac{\pi}{2}} 3\,\sin(2x)\,\mathrm{d}x = [-\frac{3}{2}\cos 2x]_0^{\frac{\pi}{2}} = \frac{3}{2} - (-\frac{3}{2}) = 3.$$

[1] The substitution $t = 2x$ has been used. Then $\mathrm{d}x = \frac{1}{2}\mathrm{d}t$. C is integration constant.