1) Simplify the expression:

$$\left(\frac{a^2}{a+2}\right)^{-1} \cdot \left(\frac{a^2-4}{a}\right)^{-1}$$

a) $\dfrac{1}{a-2}$ b) $\dfrac{a^2-4}{a+2}$ c) $\dfrac{1}{a(a-2)}$ d) $\dfrac{1}{a+2}$

2) Solve the inequality:
$$|2x-5| \leq 3.$$

a) $x \in< -5, 4 >$,   b) $x \in< 0, 3 >$,
c) $x \in< 3, 5 >$,   d) $x \in< 1, 4 >$.

3) Solve the following system of equations (for all couples of x and y):

$$\begin{aligned} x - y &= 1 \\ x^2 - 2y - 2 &= 3 \end{aligned}$$

a) $[5; 4], [0; 1/2]$ b) $[3; 2], [-1; -2]$
c) $[3; 2]$ d) "there is no solution".

4) Find solution (in real numbers) of the following equation:

$$\sqrt{x} - \frac{3-x}{\sqrt{x}} = 0.$$

a) $x = 1$ b) $x = 0$ c) $x = 3/2$ d) $x = 4$.

5) The circle given by equation $x^2 + y^2 + 8x - 6y + 9 = 0$ has its center at point S and radius r. Find S and r.

a) $S = [4; -3]$, $r = 3$ b) $S = [-4; 3]$, $r = 3$
c) $S = [4; -3]$, $r = 4$ d) $S = [-4; 3]$, $r = 4$.

6) The surface area of a cylinder with diameter $12\ cm$ is $96\pi cm^2$. Compute the volume of this cylinder.

   a) $96\pi cm^3$ b) $72\pi cm^3$ c) $64\pi cm^3$ d) $148\pi cm^3$.

8) Find the equation of the line intersecting points $A = [1,0]$ and $B = [2,2]$.

   a) $y = x - 1$ b) $y = 2x - 1$
   c) $y = 2x - 2$ d) $y = 3 - x$.

9) What is the chance (in percent rounded down in whole numbers) that the number you will get as a sum of throw of two six sided dices is $5, 6, 7$ or 8.

   a) 30% b) 55%
   c) 60% d) 40%.

10) In recent years the number of people living in a small city was rising by 3% per year. Now at 2023 there is 5500 people living in the town and the magistrate needs to plan how many will be living there 3 years later if the grow rate remains the same. Which of these possibilities is most precise guess to how many people will be in town in 2026?

   a) 5509 b) 4750
   c) 6010 d) 6535.

Solution:

1)
$$\left(\frac{a^2}{a+2}\right)^{-1} \cdot \left(\frac{a^2-4}{a}\right)^{-1} = \left(\frac{a+2}{a^2}\right) \cdot \left(\frac{a}{a^2-4}\right) = \frac{a(a+2)}{a^2(a^2-4)} = \frac{1}{a(a-2)}$$

2) Absolute value change the sign if its argument is negative so we have

$$\text{for} \quad 2x - 5 \geq 0 \quad \text{is} \quad |2x-5| = 2x - 5 \leq 3,$$
$$\text{we have} \quad 2x \leq 8 \quad \text{and then} \quad x \leq 4,$$
$$\text{for} \quad 2x - 5 < 0 \quad \text{is} \quad |2x-5| = 5 - 2x \leq 3,$$
$$\text{we have} \quad 2x \geq 2 \quad \text{and then} \quad x \geq 1.$$

Taking results together we see that $x \in < 1, 4 >$
(also sometimes written as $x \in [1, 4]$).

3) From the first equation we have

$$x = y + 1 \quad \text{inserting into second equation we get}$$
$$y^2 + 2y + 1 - 2y - 2 = 3$$
$$y^2 = 4 \quad \text{has solutions} \quad y = 2, \quad y = -2.$$

computing appropriate values of x from first equation gives $[3, 2], [-1; -2]$.

4)
$$\sqrt{x} - \frac{3-x}{\sqrt{x}} = 0 \Rightarrow \sqrt{x} = \frac{3-x}{\sqrt{x}} \Rightarrow x = 3 - x \Rightarrow x = 3/2.$$

5) We will "complete the square for x and y"

$$x^2 + y^2 + 8x - 6y + 9 = 0$$
$$\Rightarrow x^2 + 8x + 16 + y^2 - 6y + 9 - 16 = 0$$
$$\Rightarrow (x+4)^2 + (y-3)^2 = 16.$$

Now from the knowledge that the equation for a general circle with center $S = [s_1; s_2]$ and radius r is $(x - s_1)^2 + (y - s_2)^2 = r^2$ we can deduce that $S = [-4; 3]$, $r = 4$.

6)

$$\begin{aligned} S &= 2\pi r^2 + 2\pi r h = 96\pi cm^2 \text{ we know that } d = 12 \rightarrow r = 6 \\ S &= 2\pi \cdot 36 + 2\pi \cdot 6 \cdot h = 72\pi + 12\pi \cdot h = 96\pi \\ &\rightarrow 12\pi \cdot h = 24\pi \rightarrow h = 2 \\ V &= \pi r^2 h = \pi \cdot 36 \cdot 2 = 72\pi \ cm^3. \end{aligned}$$

7)

$$a + b + c = (b - d) + b + (b + d) = 15 \rightarrow 3b = 15 \rightarrow b = 5.$$
$$a \cdot b \cdot c = (b - d) \cdot b \cdot (b + d) = 80 \rightarrow (5 - d) \cdot 5 \cdot (5 + d) = 80,$$
$$\rightarrow 125 - 5d^2 = 80 \rightarrow d^2 = 9 \rightarrow d = 3.$$

So we know that $b = 5$ and $d = 3$ then $a = b - d = 2$ and $c = b + d = 8$.

8) We are looking for a equation of the line $y = kx + q$, so we need to find numbers k and q such that the line goes through both points. Inserting points A and B into the equation (i.e. $x = 1, y = 0$ and $x = 2, y = 2$) we get

$$y = kx + q \rightarrow 0 = x + q \text{ and } 2 = 2x + q$$
$$q = -x \rightarrow 2 = 2x + (-x) \Rightarrow x = 2 \rightarrow q = -2.$$

9) There is $1/6$ chance to get $1, 2, 3, 4, 5, 6$ on first and second dice. So each combination has $1/36$ chance. There are 4 ways how to get 5 ($1 + 4, 2 + 3, 3 + 2, 4 + 1$), 5 ways to get 6, 6 ways to get 7 and 5 ways to get 8. So together we have 20 out of 36 chance to get number between 5 and 8. And $20/36 = 5/9 = 55.5$ percent.

10)
$$5500 \cdot (1,03)^3 = 6009,9985 \sim 6010.$$