MASARYK INSTITUTE OF ADVANCED STUDIES

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SPECIMEN OF ADMISSION TEST

Bachelor´s degree programme – Academic Year 2021/22 - Mathematics

1) Simplify the expression:

$$\left(\frac{6a}{6-3a}+\frac{2a}{a+2}+\frac{6a}{a^2-4}\right):\frac{2}{2-a}$$

a) $\frac{1}{a-2}$ **b)** $\frac{-1}{a+2}$ **c)** $\frac{a}{a+2}$ **d)** $\frac{a}{a-2}$

2) Determine, how many integers is the solution of the following inequality.

$$|2x-4|\leq 7$$

a) 5 **b)** 6 **c)** 7 **d)** 8

3) Solve, for x and y, the following system of equations:

$$x^2+y^2+3x=4$$
$$x+2y-4=0$$

a) $\left[\frac{4}{5};\frac{12}{5}\right],[0;2]$ **b)** $\left[-\frac{4}{5};\frac{12}{5}\right],[0;2]$ **c)** $[-4;0],[0;2]$ **d)** $[-4;0],[0;-2]$

4) Determine all the solutions of the equations located in the interval $(-\pi;\pi)$

$$\sin x+\sin 2x=0$$

a) $x_1=0;\ x_2=\frac{5\pi}{6};\ x_3=\pi$ **b)** $x_1=0;\ x_2=\frac{2\pi}{3};\ x_3=\frac{5\pi}{6}$

c) $x_1 = -\frac{5\pi}{6};\ x_2 = 0;\ x_3 = \frac{5\pi}{6}$

d) $x_1 = -\frac{2\pi}{3};\ x_2 = 0;\ x_3 = \frac{2\pi}{3}$

5) Calculate the area of isosceles triangle that is inscribed in a circle. The diameter of the circle is 12 cm.

a) 27 cm^2 **b)** $\frac{81}{2}$ cm^2 **c)** $27\sqrt{3}cm^2$ **d)** $\frac{81\sqrt{5}}{5}$ cm^2

6) The surface area of a cylinder with diameter 8 *cm* is $80\pi\ cm^2$. Determine the volume of this cylinder.

a) $128\pi\ cm^3$ **b)** $96\pi\ cm^3$ **c)** $64\pi\ cm^3$ **d)** $56\pi\ cm^3$

7) The edges of cuboid *a, b, c* are first three terms of an arithmetic progression. The sum of edges *a + b + c* is 24 cm and the volume of the cuboid is 312 cm^3. Determine the first term *a* and difference *d* of the arithmetic progression.

a) *a* = 3 cm; *d* = 5 **b)** *a* = 4 cm; *d* = 4 **c)** *a* = 5 cm; *d* = 3 **d)** *a* = 6 cm; *d* = 2

8) Find the equation of a line that passes through the points A[-3;1] and B[2;2].

a) $x + 5y - 2 = 0$ **b)** $x - 5y + 8 = 0$ **c)** $5x - y + 16 = 0$ **d)** $5x + y + 14 = 0$

9) There are 8 kinds of ice cream on offer in a sweet-shop. You can choose between two cone types (normal or sweet). You can also choose one or two out of five different decorating toppings. How many ice cream combinations can you possibly buy (in that sweet-shop)?

a) 400 **b)** 240 **c)** 160 **d)** 80

10) The population of a small town decreases every year by 5%. 7 220 people lived here at the end of 2020. How many people lived here at the end of 2018?

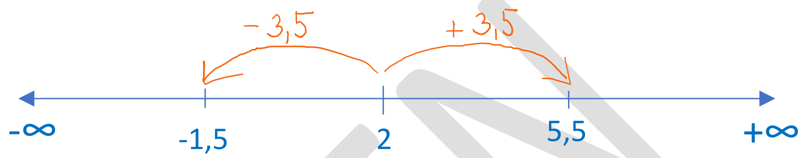
a) 7 942 **b)** 7 960 **c)** 8 022 **d)** 8 000

Solution: 1c; 2c; 3b; 4d; 5c; 6b; 7a; 8b; 9b; 10d

Solution procedures:

1) $\left(\frac{6a}{6-3a}+\frac{2a}{a+2}+\frac{6a}{a^2-4}\right):\frac{2}{2-a}=\frac{-6a(a+2)+3*2a(a-2)+3*6a}{3(a+2)(a-2)}\cdot\frac{2-a}{2}=$
$\frac{-6a^2-12a+6a^2-12a+18a}{3(a+2)}*\frac{-1}{2}=\frac{-6a}{3(a+2)}*\frac{-1}{2}=\frac{-a}{a+2}*\frac{-1}{2}=\frac{a}{a+2}$ $\quad a\neq\pm 2$

2) $|2x-4|\leq 7$



$2|x-2|\leq 7$
$|x-2|\leq 3{,}5$
$zero\ point: x-2=0\rightarrow x=2$

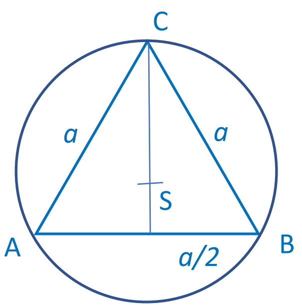
The distance from the zero (turning) point is less than 3,5 or is equal to 3,5
$x\in\langle -1{,}5;5{,}5\rangle \rightarrow$ $x\in\{-1;0;1;2;3;4;5\}$ $\rightarrow$ 7 integers

3) $x^2+y^2+3x=4$
$x+2y-4=0 \quad\rightarrow\quad x=4-2y$

$(4-2y)^2+y^2+3(4-2y)=4$
$16-16y+4y^2+y^2+12-6y=4$
$5y^2-22y+24=0$
$$y_{1,2}=\frac{22\pm\sqrt{22^2-4*5*24}}{2*5}=\frac{22\pm 2}{10}$$
$$y_1=2;\ y_2=\frac{12}{5}\ \rightarrow\ x_1=0;\ \ x_2=\frac{-4}{5}\ \rightarrow\ [0;2];\ \left[\frac{-4}{5};\frac{12}{5}\right]$$

4) $\sin x+\sin 2x=0$
$\sin x+2sinx*cosx=0$
$\sin x\cdot(1+2cosx)=0$
$\sin x=0\quad or\quad 1+2cosx=0\ \rightarrow \cos x=-\frac{1}{2}$ in the interval $(-\pi;\pi)$
$x_1=0$ $\qquad x_2=-\frac{2\pi}{3};\ x_3=\frac{2\pi}{3}$

5)



$|CS|=r=\frac{2}{3}t_a=\frac{2}{3}v_a=6cm\ \rightarrow\ v_a=9cm$

$9^2=a^2-\left(\frac{a}{2}\right)^2$
$81=\frac{3a^2}{4}\ \rightarrow\ a^2=\frac{4\cdot 81}{3}=108$
$a=\sqrt{108}=\sqrt{36*3}=6\sqrt{3}$

$$S = \frac{a \cdot v_a}{2} = \frac{6\sqrt{3} \cdot 9}{2} = 27\sqrt{3}\ cm^2$$

6) $S = 2\pi r^2 + 2\pi r v = 80\pi\ cm^2$ and $d = 8\ cm \rightarrow r = 4\ cm$
$S = 2\pi * 16 + 2\pi * 4 * v = 80\pi$
$32\pi + 8\pi v = 80\pi$
$8\pi v = 48\pi \rightarrow v = 6\ cm \rightarrow V = \pi r^2 v = \pi * 16 * 6 = 96\pi\ cm^2$

7) three terms of an arithmetic progression: $x - d; x; x + d$
$a + b + c = x - d + x + x + d = 24$
$3x = 24 \rightarrow x = 8$
$(x - d) * x * (x + d) = 312$
$(8 - d) * 8 * (8 + d) = 312$
$(8 - d) * (8 + d) = 39$
$64 - d^2 = 39$
$25 - d^2 = 0$
$(5 - d) * (5 + d) = 0$
$d_1 = 5;\ d_2 = -5 \rightarrow a = 3\ cm; b = 8\ cm; c = 13\ cm \rightarrow a = 3cm; d = 5$

(The second solution is *a= 13 cm; d = -5* and gives the same side lenghts)

8) A[-3;1] and B[2;2] gives the direction vector $\vec{u} = (5; 1)$ and
the normal vector $\vec{n} = (1; -5)$

the equation is: $x - 5y + c = 0$
substitute A[-3;1] : $-3 - 5 * 1 + c = 0 \rightarrow c = -2$
$x - 5y - 2 = 0$

9) one topping or two out of five toppings

$$8 * 2 * 5 \quad + \quad 8 * 2 * \binom{5}{2} = 80 + 16 * 10 = 80 + 160 = 240$$

10) $x * 0,95 * 0,95 = 7220$
$x * 0,95^2 = 7220$
$x = 7220 : 0,95^2$
$x = 8000$