MASARYK INSTITUTE OF ADVANCED STUDIES

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SPECIMEN OF ADMISSION TEST

Master´s degree programme – Academic Year 2021/22

1) Solve the matrix equation X·A = B for the following matrices

$$A=\begin{pmatrix}1 & 0 & -1\\ 2 & -1 & 1\\ 1 & 1 & -2\end{pmatrix};B=\begin{pmatrix}1 & 1 & 1\\ 1 & 2 & 0\\ 0 & 2 & 2\end{pmatrix}$$

a) $\frac{1}{2}*\begin{pmatrix}0 & 3 & 1\\ -4 & 3 & 1\\ -2 & 1 & -1\end{pmatrix}$

b) $\frac{1}{2}*\begin{pmatrix}0 & -4 & -2\\ 3 & 3 & 1\\ 1 & 1 & -1\end{pmatrix}$

c) $\frac{1}{2}*\begin{pmatrix}-9 & 3 & 5\\ -11 & 3 & 7\\ -16 & 4 & 8\end{pmatrix}$

d) $\frac{1}{2}*\begin{pmatrix}-9 & -11 & -16\\ 3 & 3 & 4\\ 5 & 4 & 8\end{pmatrix}$

2) Find the equation of a tangent line to the following function

$$y=\ln\left(2-x^2\right) \quad \text{at} \quad x=1$$

a) $y=2-x$

b) $y=2-2x$

c) $y=x-1$

d) $y=2x+2$

3) Calculate the area under the curve of the following function

$y = e^{2x-1} \in \textit{the interval} \langle 0,1 \rangle$

a) $\frac{e^2 - 1}{e}$

b) $\frac{e^2 - 1}{2e}$

c) $\frac{e - 1}{2}$

d) $e - 1$

4) A rational consumer is trying to attain an optimal consumption basket: Her income is $1,000, the price of good X is $100, the price of good Y is $200. The marginal rate of substitution in consumption of good Y for good X equals 1.5. This consumer maximizes the total utility but she is also restricted by the income budget. She decides

a) To increase the amount of good Y (2 units), decrease in good X (1 unit) → to increase in the total utility
b) To increase the amount of good X (2 units), decrease the amount of good Y (1 unit) → to increase the total utility
c) To increase the amount of good X (2 units), decrease the amount of good Y (1 unit) → to increase the income
d) To increase the amount of good Y (2 units), decrease the amount of good X (1 unit) → to increase the total expenditures

5) Bicycles are manufactured using an automated production line with $100,000 annual fixed costs and $5 unit costs and a $50 selling price. What is the lowest price acceptable for the firm in the short run?

a) $50 selling price
b) $40 selling price
c) $5 selling price
d) $0 selling price

6) Our economy has a negative output gap. Which of these would you expect in a developed EU economy?

a) The inflation increases, so does the economy output, there is high unemployment rate
b) The inflation falls, so does the economy output, there is high unemployment rate
c) The inflation is stable, so is the economy output, there is low unemployment rate
d) The inflation is falling, economy output is growing, there is low unemployment rate

7) A company without debts owned by its founder gets the opportunity to take out a loan. Its return on assets (ROA) is 10%, the interest rate is 4%, the corporate income tax rate is 20%. When the debt reaches 20% of total assets, the owner can expect

a) a decrease in the return on equity (ROE) while the return of assets (ROA) remains unchanged
b) a decrease in the return on assets (ROA) while the return of equity (ROE) remains unchanged
c) an increase in the return on assets (ROA) while the return of equity (ROE) remains unchanged
d) an increase in the return on Equity (ROE) while the return of assets (ROA) remains unchanged

8) Determine the **mean value** of the random variable $\mu_{10} = \mathbf{E(X)}$ from the random vector **(X, Y)**, which is given in Table 1.

Table 1

X\Y	**1**	**2**
2	0.4	0.2
2	0.2	0.2

a) 0.25
b) 0.80
c) 1.20
d) 2.00

9) Determine the **variance** of the random variable $\mathbf{v_{02} = D(Y)}$ from the random vector **(X, Y)**, which is given in Table 1.

a) 0.024
b) 0.512
c) 0.240
d) 1.200

10) Find the constant **c** so that the function:

$$f(x,y)=\begin{cases} c\times\dfrac{x^2}{1+y^2}\ for\ values\ 2\le x\le 3\,;0\le y\le 1 \\ 0\ for\ other\ values \end{cases}$$

is the **probability density** of some random vector (X, Y).

a) $\frac{1}{2}\pi$

b) $\frac{3}{4}\pi$

c) $\frac{12}{19\pi}$

d) $\frac{2}{5\pi}$

Solution: 1c; 2b; 3b, 4b; 5c; 6b; 7d, 8d; 9c; 10c.

Solution procedures

Mathematics

Ad 1) $X*A*A^{-1} = B*A^{-1}$

$X = B*A^{-1}$

Gauss elimination for the left matrix which shows the inverse matrix on the right side

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 2 & -1 & 1 & 0 & 1 & 0 \\ 1 & 1 & -2 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 3 & -2 & 1 & 0 \\ 0 & 1 & -1 & -1 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 3 & -2 & 1 & 0 \\ 0 & 0 & 2 & -3 & 1 & 1 \end{array}\right) \sim$$

$$\left(\begin{array}{ccc|ccc} 2 & 0 & 0 & -1 & 1 & 1 \\ 0 & -2 & 0 & 5 & -1 & -3 \\ 0 & 0 & 2 & -3 & 1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{-1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 & \frac{-5}{2} & \frac{1}{2} & \frac{3}{2} \\ 0 & 0 & 1 & \frac{-3}{2} & \frac{1}{2} & \frac{1}{2} \end{array}\right) \qquad A^{-1} = \frac{1}{2}\begin{pmatrix} -1 & 1 & 1 \\ -5 & 1 & 3 \\ -3 & 1 & 1 \end{pmatrix}$$

X = B* A^{-1} = = $\frac{1}{2}\begin{pmatrix} -9 & 3 & 5 \\ -11 & 3 & 7 \\ -16 & 4 & 8 \end{pmatrix}$

Ad 2) $x=1 \rightarrow y=\ln(2-1^2)=\ln 1=0$

equation of tangent line at poit $(x_0 ; y_0)$ is $y-y_0=k*(x-x_0)$ where slope $k=y´(x_0)$

$$y´=\frac{1}{2-x^2}*(-2x)=\frac{-2x}{2-x^2} \quad \text{and } k=y´(1)=\frac{-2}{1}=-2$$

$$y-0=-2*(x-1)$$
$$y=-2x+2=2-2x$$

Ad 3) $S=\int_0^1 e^{2x-1}dx=\frac{1}{2}\int_0^1 e^{2x-1}*2dx=\frac{1}{2}\int_{-1}^1 e^t\,dt=\frac{1}{2}[e^t]_{-1}^1=\frac{1}{2}*\left(e-\frac{1}{e}\right)=\frac{e^2-1}{2e}$

Substitution $t=2x-1 \rightarrow dt=2dx$

Statistics

Ad 8) We start from the relation for the first initial moment of the random variable X:

$$\mu_{10}=\mu_x=E(X)=\sum_i x_i \times p_1(x_i|)$$

Then:

$X \backslash Y$	**1**	**2**	$p_1(x_i)$	$x_i \times p_1(x_i)$
2	0.4	0.2	0.6	1.2
2	0.2	0.2	0.4	0.8
x	x	x	1	**E(X) =2**

Ad 9) We start from the relation for the second central moment of the discrete random variable Y:

$$v_{02} = \sigma_y^2 = \sum_j \left(y_j - E(Y)\right)^2 \times p_2(y_j)$$

Then:

$X \backslash Y$	**1**	**2**	**x**
2	0.4	0.2	x
2	0.2	0.2	x
$p_2(y_j)$	0.6	0.4	1
$y_j \times p_2(y_j)$	0.6	0.8	E(Y) = 1.4
$(y_j - \mu_j)^2 \times p_2(y_j)$	0.096	0.144	**D(Y) = 0.240**

Ad 10) The probability density properties are:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x, y)\,dx\,dy = 1$$

Then:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} c\frac{x^2}{1+y^2}\,dx\,dy = 1$$

$$c.\int_2^3 dx \int_0^1 \frac{x^2}{1+y^2}\,dy = 1$$

$$c.\int_2^3 dx \left[x^2 . \operatorname{arctg} y\right]_0^1 = 1$$

$$c.\int_2^3 \frac{\pi}{4} x^2\,dx = 1$$

$$\frac{\pi}{4} c.\left[\frac{x^3}{3}\right]_2^3 = 1$$

$$\frac{\pi}{4} c.\left(9 - \frac{8}{3}\right) = 1$$

$$c = \frac{12}{19\pi}$$